Cowichan Valley School District **Numeracy**FRAMEWORK







The Cowichan Valley School District recognizes, and gives thanks, that we work, live, and play on the traditional lands of the Coast Salish peoples, specifically the lands of the Lake Cowichan, Penelakut, Halalt, Lyackson, Stz'uminus, Malahat, and Quw'utsun peoples. Numeracy is the ability to understand and apply mathematical concepts, processes, and skills to solve problems in a variety of contexts. BC Curriculum



This document is the result of many educators who have partnered to create a plan that empowers all learners with numeracy skills. Numeracy is everywhere; it is our collective responsibility.

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# Rationale

After extensive consultation with our community, including local First Nations, parents, community partners, staff, and students, the Cowichan Valley School District developed a four year strategic plan. Through this process, we clarified our collective mission:

Our students are agile and prepared to transition to a future of their choosing.

In addition, our district leadership identified our north star:

### Equitable outcomes for learners.





As we continue our journey for truth and reconciliation, we first acknowledge inequities of outcomes for Indigenous learners AND we commit to working together with nuts'a'maat sqwaluwun kw tst yaa yus (one heart, one mind, and one thought) to address the inequities of outcomes. Our reflective journey identified lower than expected achievement in the areas of literacy and numeracy. In response, our district has engaged in the development of a Numeracy Framework that provides shared resources and understandings to facilitate learner growth.

All learners in the Cowichan Valley School District deserve an educational system that partners with families and community to empower them with skills, knowledge, and ways of being that will allow them to create a future of their choosing.





## **Purpose**

The purpose of our Numeracy Framework is to provide our educational community with opportunities to design practice that supports learners to become numerate citizens.

The Cowichan Valley School District Numeracy Framework is based upon current, global research and reflects the collective effort of teachers, support staff, and administrators. It is designed to share concrete examples of K-12 instruction.



Mathematics, rightly viewed, possesses not only truth but supreme beauty.

Bertrand Russell

## **Shared Beliefs**

Our Numeracy Framework is based upon shared beliefs that are reflected in all practices across our District.

### First Peoples Principles/Indigenous Perspectives

Indigenous ancestors have provided wisdom that guides our work with our xe' xe' smun'eem (sacred children). Indigenous perspectives and content are integral to our learning journey.

→ LEARN MORE

### **Culture of Care**

We recognise the critical importance of healthy relationships in safe, inclusive communities.

### **Core Competencies**

Learning experiences should integrate the following:

- Communication (knowledge, skills, processes, and dispositions associated with interacting with others)
- Thinking (combining concepts and content to transform into new understandings, habits of the mind, and metacognitive awareness)
- Personal and Social (abilities related to students' identity in the world as individuals and as members of community and society)

### Authentic engagement

Applied learning and authentic real-world experiences contribute to deeper learning for all ages.







# **First Peoples Principles of Learning**



A succinct expression of the shared wisdom of Indigenous Knowledge-keepers, scholars, and educators within British Columbia's First Peoples communities is captured within the "First Peoples Principles of Learning."

building on what students are already familiar with (both abstract knowledge and concrete knowledge)

exploring and building on students' interests (e.g., asking learners about what is important to them as a way to identify what context will prove meaningful to them as a basis for learning mathematics)

presenting mathematics problems of various sorts in varied ways (e.g., visual, oral, role-play, and experiential problems as well as word and symbol problems)

stimulating students' innate curiosity and desire to explore

communicating a positive and enthusiastic attitude toward mathematics (e.g., being willing to take risks and make mistakes and encouraging students to do the same)



promoting and rewarding perseverance (e.g., giving necessary time for difficult problems and revisiting them on multiple occasions)

encouraging students to reflect on and be explicit about their own thinking processes and the transformations in their own understanding.

### Source: FNESC, Math First Peoples Teacher Resource Guide, 2020

Our commitment to First Peoples' Principles is integrated in the collective response to supporting learners. "Data" does not focus our minds on judgement, but is seen as part of our learners' truth and story in that moment. The knowledge gained by collectively checking on all learners' reading development allows us to ensure each student can develop numeracy skills, while honouring their story, and communicating with families about strengths and next steps.



In This Section (quick links) Pedagogy & Ourselves

- Numeracy.
- Mathematics
- Number Sense 5 Strands of Proficiency
- - Value of Play.

# Foundational Understandings

This framework is based on foundational understandings that provide a common language to support professional learning and collaboration.

## Knowledge of Subject, Students and Self

The systems, structures and foundational understandings within this framework support successful mathematics education. A teacher's positive relationship with mathematics is key for students' success. These three areas support professional confidence and positive mathematics identity:

### KNOWLEDGE **OF** MATHEMATIICS

Teachers of mathematics come from a variety of backgrounds. Proficiency with conceptual and procedural knowledge of curriculum and the foundational skills of those learning standards is essential. Fluency with math vocabulary is also key.



### KNOWLEDGE **OF** STUDENTS

Understanding student strengths is essential to success. Triangulating observations, conversations and student products, provides a fulsome knowledge of a student's conceptual understanding, procedural fluency, and disposition (math identity and mindset).

YouCubed ssessment in a Thinking ssroom, P. Liljedahl (video)

### KNOWLEDGE **OF**IINSTRUCTIIONAL PRACTIICES

Knowledge of mathematics paired with knowledge of students, enables flexible and responsive instructional practices such as planning and assessment. Universal Design for Learning and Concrete-Representational-Abstract (CRA) are approaches that support this design.





Do not worry about your difficulties in Mathematics. I can assure you mine are still greater. Albert Einstein Mathematics and numeracy are not the same, though they draw on the same body of knowledge.

Numeracy

Numeracy is the ability to understand and apply mathematical concepts, processes, and skills to solve problems in a variety of contexts.

BC Curriculum

### Numeracy is embedded in all disciplines



ADST:

- calculate square footage
- budgeting materials and labour

Arts Education:

- represent perspective in artistic creations
- use timing to play or compose music

English Language Arts:

- plot story events on graphs or timelines
- interpreting statistics in a news report

### Mathematics:

- making calculations when problem solving
- determining implications of interest rate changes

### Science:

skills and knowledge

Understanding of a specific situation or

Confidence, willingness, awareness

- estimate and take measurements during experiments
- use models to represent systems, scientific structures or processes



Social Studies:

- represent events on a timeline
- interpreting numerical data

Physical & Health Education:

- create and track progress for a fitness plan
- calculate percentage of food intake



## **Mathematics**

Mathematics is the study of numbers, quantities, data, shape and space, and their relationships.

### Number sense

- is an innate ability that allows individuals to understand and work with numbers intuitively.
- develops gradually through exposure to numbers and mathematical experiences.
- is fundamental to mathematical proficiency and serves as a foundation for more advanced mathematical concepts.



Just as phonemic awareness is a prerequisite for learning phonics and becoming a successful reader, developing number sense is a prerequisite for succeeding in mathematics." Sousa (2015)

## **Computational Fluency**

- refers to the ability to perform calculations accurately, efficiently, and flexibly.
- extends beyond rote memorization of facts and includes a deep understanding of number relationships and strategies.
- develops through practice, repeated exposure to mathematical problems, and a focus on efficiency and accuracy.
- allows individuals to solve mathematical problems efficiently, freeing up cognitive resources for more complex tasks.



Real fluency is the ability to select efficient strategies; to adapt, modify, or change out strategies; and to find solutions with accuracy. Bay-Williams (2022)





LEARN MORE

**Traditionalist** 

<u>/ie</u>w: by Recovering

by Make Math Moments

## **Concrete Representational Abstract (CRA) Model**

The Concrete – Representational – Abstract (CRA) model is a simple, clear, three-part model for comprehensive mathematics planning and instruction, equally relevant in Kindergarten as it is in Grade 12.

The CRA model is evidence-based. It blends the BC curricular competencies and content, as well as aspects of both direct instruction and discovery-learning practices. CRA leads to a deeper and lasting understanding of mathematics concepts. It is best implemented as three layers of learning, not a linear process.



CRA is not linear and the three layers are used in different order and proportions, depending upon the lesson. All three, though, are part of any lesson or unit of study.





Each stage of the CRA approach should be taught using explicit, scaffolded instruction, often summarised as I Do , We Do, You Do: demonstration, modelling, and guided practice followed by independent practice, and immediate feedback.

"Sometimes there are too many questions on a page of math and it overwhelms me."
 Grade Eight Student



## **Planning Instruction within a CRA Model**

Conceptual understanding is built through the C and R layers, and is key for long-term memory and future learning. The A often refers to procedural understanding.



Students use hands, keys, and two items of choice to measure widths of a doorway, desk, bookshelf. Lived experiences promote memory and understanding. Vocabulary and concept of width and measurement practiced.

5+2=

Students gather on a staircase with a landing. The landing represents zero. Volunteer students represent financial exchange of earning and borrowing to understand the concept of negative numbers. Beyond a hook, this concretely makes meaning of the concept.

Bread is halved then halved again repeatedly. Then students imagine this continuing beyond what can be physically cut, to demonstrate the "limit" that zero will never be reached. Physical demonstrations and story/paradox builds concept of something that cannot occur. Measurement results are recorded on a table with a tally. Students compare results. Class examples used to compare and discuss different results. Reinforces vocabulary and concept of width, measurement, table and tally.

Staircase exercise is reinforced using number lines, thermometers, and pictures of money.

This is review and scaffolding deeper understanding of integers and financial literacy concepts and related vocabulary.

Graphs are used to visually represent continuity, derivatives, and integrals by teacher, as a group, then independent practice (I do, We do, You do model) Explicit key vocabulary taught and used throughout by teacher and students: vocabulary knowledge represents conceptual knowledge. Tally tables are revised adding number symbols of quantities. Class and student comparisons use symbols = and ≠ to compare equalities/inequalities. Reinforces symbols and symbolic communication of numbers, equal, and unequal.

Visual number line, thermometer, and money exercises also represented though numbers and symbols.

Reinforces symbolic communication of operations with integers. Later lessons expand concept by modelling budgets through both paper and Excel spreadsheets.

Algebraic expressions (numbers, letters, and other symbols) represent the graphic representations using all appropriate symbols. Oral communication uses technical language in explaining the math. Fulsome understanding is demonstrated when students communicate through vocabulary, visuals, and abstract algebraic expressions.

## **Concrete (Virtual) Manipulatives**















## **Examples of Classroom Assessment**

Mathematics teaching incorporates the three CRA layers when exploring areas of mathematics content.

Utilising CRA as a guide for comprehensive teaching that builds deep understanding, long-term memory, and -ultimately- computational fluency, also supports effective assessment FOR and OF learning. Triangulation of conversations, observations, and products, is inextricably embedded within the CRA layers.

Using CRA as a lens for assessment as well as instruction, results in more accurate and comprehensive understanding of student proficiency with Learning Standards. It allows us to identify when students truly understand math concepts (concrete & representational) and know how to communicate those concepts symbolically (abstract). Proficiency requires all three layers to ensure solid conceptual understanding and the ability to communicate through numbers and symbols.



### Abstract can't stand alone

If a student is only taught and assessed through the Abstract (numbers symbols), they could memorise an algorithm or follow steps without understanding the concept. This may get them through that moment, but may not be successful when problems become more complex or the context varies. Without a clear and early foundation of understanding, a student could "do well" in math for years, then struggle later.



- <u>Nix the Tricks</u> free download with examples
- Math with Bad Drawings article with examples

## **Concepts Versus Applications**

Concepts and applications are distinct components of a balanced, comprehensive understanding of mathematics. Conceptual understandings address why the mathematics work when they are applied to real-world, problem-solving situations.



Adapted from Strategies for Mathematics Instruction and Intervention

### **Conceptual Understanding (Number Sense)**

Conceptual understanding refers to "relational" understanding. This is the "whats" and "whys" of mathematics.



Students with conceptual understanding:

- know more than isolated facts and methods
- organise their knowledge into a coherent whole,
- learn new ideas by connecting them to prior knowledge.

Learning with understanding makes facts and methods easier to remember and use.



Model of Conceptual Understanding Everyday Mathematics



### **Computational Fluency**

Computational fluency refers to the understanding of when and how to use procedures. Students with procedural fluency can:

• deepen their understanding of mathematical ideas or solving mathematics problems.

## Problem Solving Skills include:

### **Strategic Competence**

Strategic competence refers to the ability to formulate mathematical problems, represent, and solve them.

A student with strategic competence can:

- come up with several approaches to a non-routine problem
- choose among different methods to suit the situation.



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Grade 2 proof for 10-5 ≠ 7 from <u>IB Education Blog</u>

3 left+ 3=6 6+3=9

Source: Dr Jennifer Suh

12+3=15

Work backwood Mork backwood Method



Government of Canada: Decolonizing math education

### Adaptive Reasoning

Adaptive reasoning refers to the capacity for logical thought, reflection, explanation, and justification.

A student with adaptive reasoning can:

- convince others that their solutions are correct
- reflect on their work.

### **Productive Disposition**

Productive disposition refers to the tendency to see sense in mathematics, perceive it as both useful and worthwhile. A student with productive disposition has:

- growth mindset
- a positive mathematical identity
- a healthy relationship with math.

C<u>lick here for</u> elaborations on each Strand of Mathematical Proficiency



## The Value of Play

Throughout a school day, approaches fluctuate between play-based teaching and structured teaching. For optimal learning, these approaches are employed at different times and for different purposes. The result is that learners receive explicit instruction as well as active learning where they practice skills through play.

In grades K-12, play has a direct, positive impact on:

- executive functioning
- self-regulation skills
- communication and
- problem-solving skills.

Play-based opportunities allow for learning through doing and the application of skills. Play may take many forms including open-ended explorations and guided challenges where the learner has the opportunity to explore or apply concepts in new ways. These lived experiences allow for a more fulsome understanding of concepts and promotes long-term memory.



Adapted from Pyle and Danniels, 2017



Educators, advocates for children's rights, and academic researchers agree that play is vital for young children. Play should have a central role in the lives of young children—from infancy into middle childhood and beyond. The BC government recognizes the importance of playbased learning. BC Ministry of Education

The acquisition of a rich, comfortable sense of number is incremental, and is enriched by play both inside and outside the classroom. Boaler (2022)



In This Section (quick links) MTSS

- Check-In for planning
- District Screeners
- Use of Data

## **District and School-Wide Systems & Structures**

The following research and evidence-based systems and structures enable the actions of the Cowichan Valley School District Numeracy Framework.

## **MTSS: Multi-Tiered Systems of Support**

MTSS is a coordinated system of academic, emotional, and behavioural supports (universal, targeted, intensive) which provides a structure for effective instruction, assessment, and support for all students. It requires the collaboration and coordinated efforts of students, school and district staff, families, and community partners in providing appropriate programs, settings, supports, and services.

### MTSS elements include:

- Universal screening of all students early in the school year
- Tiers of interventions that can be amplified in response to identified needs
- Ongoing data collection and continual assessment
- School-wide approach to expectations and supports
- Involvement of supporting adults (guardians, school-based teams, specialists...)





In This Section (quick links) Assessment Spiralling & Interleaving

- Resources

## **Instructional Systems** and Considerations

### Numeracy Universal Screener

The Kindergarten to Grade 9 Numeracy Screening Tool is a series of assessments developed by teachers. These assessments and the procedures were developed through working groups of teachers and district staff. The assessments are intended to provide useful information to inform instruction and supports.

### Links to Grade by grade screeners of critical skills



### **Learning Inventory Data Collection**



Ongoing Assessment for Learning and Assessment of Learning: Authentic assessment of a student's proficiency with the Learning Standards (curricular competencies and content) is gained through multiple methods and tools that best fit the student and the context. Triangulating evidence from observations, conversations, and products ensures a more accurate understanding of a student's

#### Triangulating in secondary classes Scenario A:

proficiency.

A student's product on an assignment suggests mastery; yet the observation of struggle indicates a developing level of proficiency with those learning standards. That observation is key evidence, providing further guidance for teaching and meaningful feedback.

#### Scenario B:

A student's proficiency with a learning standard may be assumed to be emerging based on a single test, but in class challenges, they demonstrate a consistent ability to apply the concepts in complex scenarios. This evidence can be clarified with conversation to add to the observed data.



### Learning Inventory Data Use

Our screeners provide data to inform instructional decisions at the classroom, school and district levels, as well as information related to every learner's journey with numeracy skills.



### Classroom Use of Data

The data shows what curricular and foundational skills learners have mastered and what skills need additional instruction.

By collaborating, teachers can identify:

- skills that should be addressed through whole class instruction
- skills in which small groups or individual students need extra support or practice.

### School Use of Data

Through PLCs, SBT, and other collaborative conversations, schools can use the data to:

- identify groups or classes of students who require
  extra supports
- focus supports where students have the greatest need
- identify student needs that have common instruction and intervention strategies (across classrooms).



### District Use of Data

At the district level, data can:

- help guide decisions related to in-service, and instructional recommendations
- identify structures, routines, and instructional materials to address learning needs that can be supported by district learning teams
- develop district-lead learning opportunities to respond to trends or emerging needs.



### **Instruction for Learners of All Abilities**

Differentiated math instruction refers to the techniques, strategies, and adaptations that will support all diverse learners to access numeracy-focused learning standards and be able to show growth and success. "We all learn in different ways and have different strengths."

High School Student

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#### Problem Solving Framework

A tool for helping students solve problems. It scaffolds students often underdeveloped critical thinking skills and helps them develop their own problem solving techniques.

### **Project-Based Learning**



For many students, it can be difficult to make a real-life connections between numeracy and their everyday lives. Through a Project-Based Learning (PBL) approach, students learn that numeracy skills are not only theoretical but practical and necessary. Students move beyond a basic understanding of concepts to the enjoyment of discovery.

### 14 Practices in Thinking Classrooms



In a thinking mathematics classroom the right task is important. When first starting to build a thinking classroom it is important that these tasks are highly engaging. The goal is to get more of our students thinking, and thinking for longer periods of time, within the context of curriculum.

Building Thinking Classrooms in Mathematics (2018)

### Creating Cultures of Math



Create the conditions for students to think deeply and own their learning. Create curiosity and stretch students while allowing them to meaningfully contribute to the collective understanding of the group. This resource connects current numeracy research with Core Competencies and self-reflection.



## **Universal Design for Learning (UDL)**

Adapted from CAST and Lambert

Universal Design for Learning (UDL) is a framework to improve and optimise teaching and learning for all people based on scientific insights into how humans learn. The three principles of UDL are:



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COWICHAN VALLEY School District

"Mathematics knows no races or geographic boundaries; for mathematics, the cultural world is one country." – David Hilbert

### **Spiralling the Curriculum**

Adapted from Make Math Moments and Interleaving in Math

### Spiralling the Math Curriculum is

- commonly referred to as "interleaving", "distributing", "spacing" or "mixing" topics.
- revisiting and reinforcing previously learned concepts over time
- repetition and frequent exposure to concepts can enhance long-term retention and understanding

When spiralling the curriculum in math class, topics are introduced in smaller chunks and then spread out over a longer period of time, instead of in units or chapters.

It is common to come back to the topic multiple times over the duration of the grade or course and go deeper each time.

### Spiralling

Throughout learning, students have the opportunity to grow in curricular competencies because they experience those competencies through a variety of content areas.



#### →) LEARN MORE

- <u>The Complete Guide to Spiralling...</u> Making Math Moments
  - Interleaving in Math (implementation tips)
- <u>Sample Year Plans</u> from Delta
- Year Planning Guide from Delta
- Grade 4 Unit Plan from Delta

On the whole, both in the laboratory and the classroom, both in adults and in children, and in the cognitive and motor learning domains, spacing leads to better performance than massing. Son & Simon, 2012)



## **Instructional Resources**

Samples of foundational skills in number sense and computational fluency identified by Cowichan Valley School District teachers.

### Primary

- ways to make 5 and 10
- one-to-one correspondence
- addition and subtraction fact fluency

### Intermediate

- place value concepts to numbers less than one
- multiplication and division fact fluency
- addition and subtraction of decimals

### Secondary

- understanding of fractions
- place value using positive and negative numbers
- linear relations

Resource documents that contain carefully curated, extensive resources related to foundational skills at each grade can be found at the links below.



### **General Recommended Resources**

- YouCubed
- Building Thinking Classrooms
- Number Talks

- BC Numeracy Network
- BC Association of Mathematics Teachers
- <u>Coast Metro Elementary Math Project</u>



## **Scanning Tools**

### Numeracy Check-In: Self-Assessment Tools



The self-assessment tool can support school professional communities to identify numeracy knowledge, expertise, strengths, and needs. These questions can also initiate an understanding of the strengths and needs of learners.

These questions are open-ended and can be used as a way to initiate a path to indepth numeracy planning. Ideally, this dialogue includes all staff who will support the learning, both directly and indirectly.

Questions are divided into the following sections:

- Culture
- Structures
- Instruction

The questions serve to support a professional process of inquiry and can be used or adapted as fits the needs of each team.



**School Based Team** 

Whole staff



### Self-Assessment Tool Multi-Disciplinary System of Support



### Culture

1. Our school demonstrates a collective belief that every student can be proficient with mathematics.

- Definitely
- We're making meaningful progress
- We will give this some thought

2.Our school values professional learning for all teachers in discipline-specific numeracy instruction and skill development.

- Definitely
- We're making meaningful progress
- We will give this some thought
- 3. Our school practices a universal design for learning approach in response to student assessment data. Definitely
  - We're making meaningful progress
  - We will give this some thought

### **Structures**

1.Our school has identified blocks of time that are protected from interruption for numeracy instruction.

- Definitely
- We're making meaningful progress
- We will give this some thought
- Describe

2.School team collectively reviews data regularly to identify whole group needs, small group instruction, and needs for intervention.

- Definitely
- We're making meaningful progress
- We will give this some thought
- Describe
- 3. Our school has numeracy tools and resources that are readily available to educators. Definitely
  - We're making meaningful progress
  - We will give this some thought
  - Describe

4. Structures are in place in our school to identify students' targeted numeracy needs, intervention opportunities are available, and the success of the interventions are monitored.

- Definitely
- We're making meaningful progress
- We will give this some thought
- Describe

#### Instruction

1. Our educators recognise common language related to mathematics and numeracy.

- Definitely
- We're making meaningful progress
- We will give this some thought
- Describe
- 2. Our educators feel confident in their knowledge of the CRA model. Definitely
  - We're making meaningful progress
  - We will give this some thought
  - Describe
- 3. Teachers are clearly aware of the continuum of skills across grade levels. Definitely
  - We're making meaningful progress
  - We will give this some thought
  - Describe
- 4. Educators consistently use scaffolding and modelling (I do, We do, You do). Definitely
  - We're making meaningful progress
  - We will give this some thought
  - Describe

5.Students whose assessments show a need for intervention (Tier 2/3) are also active members of the Tier 1 community.

- Definitely
- We're making meaningful progress
- We will give this some thought
- Describe
- 6. Mathematics educators provide explicit instruction in the following areas:. conceptual understanding
  - procedural fluency
  - strategic competence
  - adaptive reasoning
  - productive disposition.

Celebrations of your school's numeracy approaches:

Hopes and wishes for your school's numeracy approaches:



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